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NON-STATIONARY SEISMIC RESPONSE OF MDOF SYSTEMS BY WAVELET TRANSFORM

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SUMMARY

A wavelet-based formulation has been presented in this paper for the stochastic analysis of a linear multi-degree-of-freedom (MDOF) classically damped system subjected to earthquake ground motion. The ground motion has been modelled as a non-stationary process (both in amplitude and frequency) using wavelets. Closed-form expressions of the moments of the instantaneous Power Spectral Density Function (PSDF) of the response have been derived and used to predict the statistics of the response peak of any desired order. For illustration of the formulation, an example torsionally coupled multistoried building has been considered along with the twenty synthetically generated time-histories corresponding to an example ground motion process. © 1997 John Wiley & Sons, Ltd.

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KEY WORDS: wavelet transform; Littlewood–Paley basis; frequency non-stationarity; modal correlation; time-dependent PSDF; ordered peak amplitudes

1. INTRODUCTION

There are several engineering problems of relevance which can be modelled as the responses of linear multi-degree-of-freedom (MDOF) systems to non-stationary excitations and thus have been of interest to the civil engineers. Most of those involve the excitation of linear, classically damped structural systems by earthquakes at their base. Non-stationarity in the earthquake ground motions arises due to the temporal variations in amplitude and frequency content. Whereas the former type of non-stationarity is associated with the zero initial start (see Reference 1) and decaying nature of the excitation, the latter type is due to the dispersion of the propagating seismic waves. The amplitude non-stationarity has been tackled in studying the response of single-degree-of-freedom (SDOF) and MDOF systems mostly by modelling the input as a uniformly modulated stationary process. Some of the popular amplitude modulating functions, also referred to in the literature as 'strength function', used for this purpose include step function by Caughey and Stumpf,[1] and Corotis and Vanmarcke,[2] boxcar function by Barnoski and Maurer,[3] staircase function by Hasselman,[4] half-sine function by Lin,[5] gamma function by Conte and Peng,[6] exponentially decaying function by Corotis and Marshall,[7] To,[8] and Borino *et al*.,[9] and piecewise linear functions by Gasparini and DebChaudhury.[10] Several other researchers, e.g. Unruh and Kana,[11] Shrikhande and Gupta,[12,13] modelled ground motion processes as 'equivalent stationary' processes. The frequency non-stationarity has however not been considered explicitly by the researchers, even though the earthquake ground motions are known to be highly non-stationary in this respect due to the arrival of body waves followed by the arrival of several modes of surface waves (e.g. see References 14–16). This

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was perhaps because the existing theory of random vibrations for linear systems could be used only if the ground motions were characterized entirely in time or frequency domain. It was thus incompatible with any characterization of the ground motions simultaneously based on time and frequency.

The recent development of wavelet analysis techniques has made it possible to represent the temporal variations of frequency content in a signal elegantly. These techniques are more versatile than the other time-frequency localizing techniques (e.g. Gabor transform, short-time Fourier transform) due to the flexible time-frequency windowing feature (see References 17 and 18). This feature allows construction of basis functions with a desired time-frequency resolution. A considerable amount of research has been carried out (e.g. see References 18–22) and several wavelet functions have been developed with specified characteristics (e.g. those by Stromberg,[23] Battle,[24] Lemarié,[25] and Meyer[22]) to suit different applications. In engineering applications, Newland[17,26,27] used Daubechies' wavelet for the analysis of vibration signals. He also developed Harmonic wavelet and devised Discrete Wavelet Transform (DWT) and Fast Wavelet Transform (FWT) computational algorithms.

In this paper, a new stochastic formulation has been proposed for the analysis of MDOF systems subjected to non-stationary excitations. A wavelet-based framework without the assumption of a specific modulating function has been considered to obtain closed-form explicit solutions of the moments of the instantaneous power spectral density function (PSDF) in a generalized form. The wavelet basis used in the paper is close to both Littlewood–Paley (L–P) basis and Harmonic wavelet basis by Newland.[26] The statistics of various ordered peaks (i.e. first largest, second largest, etc.) in various responses of a MDOF system are calculated by a hybrid approach based on the first passage formulation of Vanmarcke[28] for a non-stationary process and the order statistics formulation of Gupta and Trifunac,[29] Gupta[30] and Todorovska *et al.*[31] The proposed formulation has been illustrated by comparing the stochastic estimates on the expected displacement and shear response profiles for different ordered peaks with those obtained through statistical simulation for an example torsionally coupled multistorey building and an example ground motion process.

2. WAVELET-MODELLING OF GROUND MOTIONS

Let us consider the ground acceleration process as a zero-mean, non-stationary process, $z(t)$, with locally Gaussian characteristics. The wavelet transform, $W_\psi z(a,b)$ of $z(t)$ with respect to the translated and dilated form, $\psi_{a,b}(t)$ $(=|a|^{-1/2}\psi((t-b)/a))$ of a real wavelet basis, $\psi(t)$, and its inversion relationship are given as follows (see, e.g. Reference 18)

$$W_\psi z(a,b)=\int_{-\infty}^{\infty} z(t)\psi_{a,b}(t)\,\mathrm{d}t, \quad a>0 \tag{1}$$

and

$$z(t)=\frac{1}{2\pi C_\psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{a^2}W_\psi z(a,b)\psi_{a,b}(t)\,\mathrm{d}a\,\mathrm{d}b \tag{2}$$

with

$$C_\psi=\int_{-\infty}^{\infty}\frac{|\hat{\psi}(\omega)|^2}{|\omega|}\,\mathrm{d}\omega<\infty \tag{3}$$

In equation (3), $\hat{\psi}(\omega)$ is the Fourier transform of the basic wavelet function $\psi(t)$ given by

$$\hat{\psi}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\psi(t)\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}t \tag{4}$$

In the convolution integral in equation (1), the parameter b has the physical significance of localizing the basis function at $t=b$ and its neighbourhood, whereas the parameter a captures the local frequency content as is

apparent from the Fourier transform of $\psi_{a,b}(t)$, i.e. $\hat{\psi}(a\omega)e^{i\omega b}$. For numerical evaluation of the above integrals, we follow a scheme similar to that by Alkemede,[32] and discretize a at $a_j = \sigma^j$ and b at $b_j = (j-1)\Delta b$ where σ and Δb are the discretization parameters. We now define the step changes at $a = a_j$ and $b = b_j$, respectively, by

$$\Delta b_j = [(b_{j+1} - b_j) + (b_j - b_{j-1})]/2 = \Delta b \tag{5}$$

and

$$\Delta a_j = [(a_{j+1} - a_j) + (a_j - a_{j-1})]/2 = \frac{a_j}{2}\left(\sigma - \frac{1}{\sigma}\right) \tag{6}$$

The discretized version of equation (2) is therefore obtained as

$$z(t) = \sum_i \sum_j \frac{K\,\Delta b}{a_j} W_\psi z(a_j, b_i)\psi_{a_j,b_i}(t) \tag{7}$$

with

$$K = \frac{1}{4\pi C_\psi}\left(\sigma - \frac{1}{\sigma}\right) \tag{8}$$

By using the Parseval's identity and a discretized version of the following theorem (see Reference 18),

$$\langle z, z\rangle = \int_{-\infty}^{\infty} z^2(t)\,dt = \frac{1}{2\pi C_\psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{[W_\psi z(a,b)]^2}{a^2}\,da\,db \tag{9}$$

we obtain the expected total energy of the process $z(t)$ as

$$\int_{-\infty}^{\infty} E[|Z(\omega)|^2]\,d\omega = \sum_i \sum_j \frac{K\,\Delta b}{a_j} E[W_\psi z(a_j, b_i)]^2 \tag{10}$$

where $Z(\omega)$ is the Fourier transform of $z(t)$. The expected energy for $z(t)$ in the frequency band corresponding to the discretized value a_j is thus given by

$$\int_{-\infty}^{\infty} E[|Z^j(\omega)|^2]\,d\omega = \sum_i \frac{K\,\Delta b}{a_j} E[W_\psi z(a_j, b_i)]^2 \tag{11}$$

Also, the instantaneous mean-square value of the process, $z(t)$ is obtained from equation (9) as

$$E[z^2(t)]|_{t=b_i} = K\sum_j \frac{E[W_\psi z(a_j, b_i)]^2}{a_j} \tag{12}$$

and the expected (instantaneous) mean-square energy for $z(t)$ in the frequency band corresponding to $a = a_j$ is given by

$$\frac{1}{\Delta b}\int_{-\infty}^{\infty} E[|Z_i^j(\omega)|^2]\,d\omega = \frac{K}{a_j} E[W_\psi z(a_j, b_i)]^2 \tag{13}$$

It may be noted that even though the above equations have been written for the ground acceleration process, $z(t)$, similar equations can also be written for any response process modelled through wavelets.

The choice of the wavelet basis function $\psi(t)$ which should be used to model $z(t)$ in the above equations primarily depends on how suitable a given basis function is for the dynamical system analysis. In this study, we consider a basis function which is a slightly modified form of the L–P basis. The L–P basis is characterized by a reasonably fast temporal decay (thus helping to capture the local temporal features) and by an excellent

frequency localization (since the Fourier transform of this basis is defined over a finite interval). The Fourier transform of this basis function may be expressed as

$$\hat{\psi}(\omega) = \begin{cases} \frac{1}{\sqrt{2(\sigma-1)\pi}}, & \pi \leqslant |\omega| \leqslant \sigma\pi \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

where σ is the scalar factor used in discretizing a. On taking the inverse Fourier transform of equation (14), we obtain

$$\psi(t) = \frac{1}{\pi\sqrt{(\sigma-1)}} \frac{\sin \sigma\pi t - \sin \pi t}{t} \tag{15}$$

It may be shown that the translated and dilated form, $\psi_{a,b}(t)$, of this wavelet basis leads to a family of mutually orthogonal functions. This helps in simplifying the input–output relationship of a linear dynamical system. Also, since the frequency bands corresponding to this basis function are non-overlapping and of finite band-widths, the input of one particular band does not affect the response in the other bands.

3. WAVELET-BASED RESPONSE OF MDOF SYSTEMS

Let us consider an n-DOF structural system subjected to the base acceleration, $z(t)$. The equations of motion of this system can be written in terms of n coupled linear equations as

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = -[m]\{\Gamma\}z \tag{16}$$

where $[m]$, $[c]$ and $[k]$, respectively, are the $n \times n$ mass, damping and stiffness matrices; $\{\Gamma\}$ is the $n \times 1$ ground displacement influence vector; and $\{x\}$ is the $n \times 1$ vector consisting of relative displacements, $x_i(t)$, $i = 1, 2, \ldots, n$ of the floor masses with respect to the ground. Assuming the system to be classically damped and on expanding the system response, $\{x(t)\}$ in terms of its normal modes, $\{\phi^{(1)}\}$, $\{\phi^{(2)}\}, \ldots, \{\phi^{(n)}\}$, as:

$$\{x(t)\} = \sum_{j=1}^{n} \{\phi^{(j)}\}\eta_j(t) \tag{17}$$

equation (16) can be transformed into following n uncoupled equations of motion, each equation describing the motion in a particular mode of vibration,

$$\ddot{\eta}_j + 2\zeta_j\omega_j\dot{\eta}_j + \omega_j^2\eta_j = -\alpha_j z; \quad j = 1, 2, \ldots, n \tag{18}$$

Here, ω_j, ζ_j, and α_j $(= \{\phi^{(j)}\}^{\mathrm{T}}[m]\{\Gamma\}/\{\phi^{(j)}\}^{\mathrm{T}}[m]\{\phi^{(j)}\})$, respectively, represent the natural frequency, damping ratio and participation factor in the jth mode.

On taking the wavelet transform of both sides of equation (17) with respect to the wavelet basis as in equations (14) and (15), we obtain the following relationship for the relative displacement response, $x_i(t)$, at the ith floor:

$$W_\psi x_i(a_p, b_q) = \sum_{j=1}^{n} \phi_i^{(j)} W_\psi \eta_j(a_p, b_q) \tag{19}$$

where $\phi_i^{(j)}$ is the ith element of the mode shape $\{\phi^{(j)}\}$. On taking expectation of the square of equation (19), we obtain

$$E[W_\psi x_i(a_p, b_q)]^2 = \sum_{j=1}^{n} \left[(\phi_i^{(j)})^2 E[W_\psi \eta_j(a_p, b_q)]^2 + \sum_{k=1, k\neq j}^{n} \phi_i^{(j)}\phi_i^{(k)} E[W_\psi \eta_j(a_p, b_q) W_\psi \eta_k(a_p, b_q)] \right] \tag{20}$$

To evaluate the terms on the right-hand side of equation (20), we substitute the wavelet expansions of $\eta_j(t)$ and $z(t)$ (as in equation (2)) in equation (18). On Fourier transforming the resulting equation, we obtain

$$\sum_q \sum_p \frac{1}{a_p} W_\psi \eta_j(a_p, b_q) \hat{\psi}_{a_p,b_q}(\omega) = \sum_q \sum_p \frac{1}{a_p} W_\psi z(a_p, b_q) H_j(\omega) \hat{\psi}_{a_p,b_q}(\omega) \tag{21}$$

where

$$H_j(\omega) = \frac{1}{(\omega^2 - \omega_j^2) + \mathrm{i}(2\zeta_j \omega \omega_j)} \tag{22}$$

is the transfer function for the relative displacement response of the oscillator in the jth mode. We now multiply both sides of equation (21) by $\hat{\psi}^*_{a_u,b_v}(\omega)$ which is the complex conjugate of the Fourier transform of $\psi_{a_u,b_v}(t)$. On using the relationship,

$$\hat{\psi}_{a_p,b_q}(\omega) \hat{\psi}^*_{a_u,b_v}(\omega) = 0 \quad \text{for} \;\; p \neq u \tag{23}$$

which can be shown to follow from equation (14), we thus obtain

$$\sum_q W_\psi \eta_j(a_p, b_q) \hat{\psi}_{a_p,b_q}(\omega) = \sum_q W_\psi z(a_p, b_q) H_j(\omega) \hat{\psi}_{a_p,b_q}(\omega) \tag{24}$$

A similar treatment for the modal response, η_k, yields

$$\sum_q W_\psi \eta_k(a_p, b_q) \hat{\psi}_{a_p,b_q}(\omega) = \sum_q W_\psi z(a_p, b_q) H_k(\omega) \hat{\psi}_{a_p,b_q}(\omega) \tag{25}$$

Multiplying equation (24) by the complex conjugate of equation (25), taking expectation on both sides, and then integrating over ω leads to (on neglecting the cross-terms associated with the co-efficients, $E[W_\psi z(a_p, b_k) W_\psi z(a_p, b_m)]$, $m \neq k$, as reasoned by Basu and Gupta[33])

$$\sum_q E[W_\psi \eta_j(a_p, b_q) W_\psi \eta_k(a_p, b_q)] = \sum_q \alpha_j \alpha_k E[W_\psi z(a_p, b_q)]^2 \int_{-\infty}^{\infty} |H_j(\omega) H_k^*(\omega)| |\hat{\psi}_{a_p,b_q}(\omega)|^2 \, \mathrm{d}\omega \tag{26}$$

For $j = k$, equation (26) leads to

$$\sum_q E[W_\psi \eta_j(a_p, b_q)]^2 = \sum_q \alpha_j^2 E[W_\psi z(a_p, b_q)]^2 \int_{-\infty}^{\infty} |H_j(\omega)|^2 |\hat{\psi}_{a_p,b_q}(\omega)|^2 \, \mathrm{d}\omega \tag{27}$$

On substituting equations (26) and (27) in equation (20), we get the expression for $E[W_\psi x_i(a_p, b_q)]^2$ in terms of the wavelet co-efficients of the input acceleration process. This expression can now be used to obtain the total expected energy of the response, $x_i(t)$ as (as in equation (10) for $z(t)$)

$$\begin{aligned} \int_{-\infty}^{\infty} E[|X_i(\omega)|^2] \, \mathrm{d}\omega = & \int_{-\infty}^{\infty} \sum_q \sum_p \frac{K \, \Delta b}{a_p} E[W_\psi z(a_p, b_q)]^2 \sum_{j=1}^{n} \Bigg[(\phi_i^{(j)})^2 \alpha_j^2 |H_j(\omega)|^2 \\ & + \sum_{k=1, k \neq j}^{n} \phi_i^{(j)} \phi_i^{(k)} \alpha_j \alpha_k \, \mathrm{Re}(H_j(\omega) H_k^*(\omega)) \Bigg] |\hat{\psi}_{a_p,b_q}(\omega)|^2 \, \mathrm{d}\omega \end{aligned} \tag{28}$$

Similarly, on using the expressions in equations (12) and (13), we get the instantaneous mean-square value of $x_i(t)$ as

$$\begin{aligned} E[x_i^2(t)]|_{t=b_q} = & K \sum_p \frac{E[W_\psi z(a_p, b_q)]^2}{a_p} \int_{-\infty}^{\infty} E[W_\psi z(a_p, b_q)]^2 \sum_{j=1}^{n} \Bigg[(\phi_i^{(j)})^2 \alpha_j^2 |H_j(\omega)|^2 \\ & + \sum_{k=1, k \neq j}^{n} \phi_i^{(j)} \phi_i^{(k)} \alpha_j \alpha_k \, \mathrm{Re}(H_j(\omega) H_k^*(\omega)) \Bigg] |\hat{\psi}_{a_p,b_q}(\omega)|^2 \, \mathrm{d}\omega \end{aligned} \tag{29}$$

and the instantaneous mean-square response energy for $x_i(t)$ at an instant $t=b_q$ in the frequency band corresponding to a_p as

$$\int_{-\infty}^{\infty} E[|X_i^p(\omega)|^2]\,\mathrm{d}\omega = \int_{-\infty}^{\infty} \frac{K\,\Delta b}{a_p} E[W_\psi z(a_p, b_q)]^2 \sum_{j=1}^{n} \left[(\phi_i^{(j)})^2 \alpha_j^2 |H_j(\omega)|^2 + \sum_{k=1,\,k\neq j}^{n} \phi_i^{(j)} \phi_i^{(k)} \alpha_j \alpha_k \,\mathrm{Re}(H_j(\omega) H_k^*(\omega)) \right] |\hat{\psi}_{a_p, b_q}(\omega)|^2 \,\mathrm{d}\omega \tag{30}$$

Even though the above equation is strictly valid in an integral sense only, it may be true in pointwise sense also provided σ is chosen close to unity. However, a popularly chosen value of σ is 2 corresponding to the 'dyadic wavelets' which are quite commonly used in wavelet analysis (e.g. see Reference 18). It may be possible to still retain the dyadic nature in a crude sense while lowering σ to close to unity, by adopting a generic form of $\sigma = 2^{c/d}$ where c and d ($\geqslant c$) are positive integers. Several experiments on wavelet modelling of the ground motions have shown that it may be quite appropriate to choose $c=1$ and $d=4$. Thus, on choosing $\sigma = 2^{1/4}$, equation (30) approximately leads to

$$E[|X_i^p(\omega)|^2] = \frac{K\,\Delta b}{a_p} E[W_\psi z(a_p, b_q)]^2 \sum_{j=1}^{n} \left[(\phi_i^{(j)})^2 \alpha_j^2 |H_j(\omega)|^2 + \sum_{k=1,\,k\neq j}^{n} \phi_i^{(j)} \phi_i^{(k)} \alpha_j \alpha_k \,\mathrm{Re}(H_j(\omega) H_k^*(\omega)) \right] |\hat{\psi}_{a_p, b_q}(\omega)|^2 \tag{31}$$

Now, on summing over all energy bands, the instantaneous PSDF at $t=b_q$ is obtained from equation (31) as

$$S_{x_i}^q = \sum_p \frac{E[|X_i^p(\omega)|^2]}{\Delta b} = \sum_p \frac{K}{a_p} E[W_\psi z(a_p, b_q)]^2 \sum_{j=1}^{n} \left[(\phi_i^{(j)})^2 \alpha_j^2 |H_j(\omega)|^2 + \sum_{k=1,\;k\neq j}^{n} \phi_i^{(j)} \phi_i^{(k)} \alpha_j \alpha_k \,\mathrm{Re}(H_j(\omega) H_k^*(\omega)) \right] |\hat{\psi}_{a_p, b_q}(\omega)|^2 \tag{32}$$

By using the partial fractions for $\mathrm{Re}(H_j(\omega) H_k^*(\omega))$ as in Gupta and Trifunac,[34,35] equation (32) is further simplified to

$$S_{x_i}^q = \sum_p \frac{K}{a_p} E[W_\psi z(a_p, b_q)]^2 \sum_{j=1}^{n} |H_j(\omega)|^2 \left[(\phi_i^{(j)})^2 \alpha_j^2 + \sum_{k=1,\;k\neq j}^{n} \phi_i^{(j)} \phi_i^{(k)} \alpha_j \alpha_k \left\{ C_{jk} + \left(1 - \frac{\omega^2}{\omega_j^2} \right) D_{jk} \right\} \right] |\hat{\psi}_{a_p, b_q}(\omega)|^2 \tag{33}$$

where C_{jk} and D_{jk} are the co-efficients given in terms of ζ_j, ζ_k and $\varrho = \omega_k/\omega_j$ as

$$C_{jk} = \frac{1}{B_{jk}} [8\zeta_j(\zeta_j + \zeta_k \varrho)\{(1-\varrho^2)^2 - 4\varrho(\zeta_j - \zeta_k \varrho)(\zeta_k - \zeta_j \varrho)\}] \tag{34}$$

$$D_{jk} = \frac{1}{B_{jk}} [2(1-\varrho^2)\{4\varrho(\zeta_j - \zeta_k \varrho)(\zeta_k - \zeta_j \varrho) - (1-\varrho^2)^2\}] \tag{35}$$

$$B_{jk} = 8\varrho^2[(\zeta_j^2 + \zeta_k^2)(1-\varrho^2)^2 - 2(\zeta_k^2 - \zeta_j^2 \varrho^2)(\zeta_j^2 - \zeta_k^2 \varrho^2)] + (1-\varrho^2)^4 \tag{36}$$

Generalizing equation (33) for any other response quantity, $r(t)$ $(=\sum_{j=1}^{n}\rho_j\xi_j(t))$, we can express the instantaneous PSDF as

$$S_r^q = \sum_p \frac{K}{a_p} E[W_\psi z(a_p,b_q)]^2 \sum_{j=1}^{n} \left[\left\{ \rho_j^2\alpha_j^2 + \sum_{k=1,\ k\neq j}^{n} \rho_j\rho_k\alpha_j\alpha_k(C_{jk}+D_{jk}) \right\} |H_j(\omega)|^2 - \frac{|\omega H_j(\omega)|^2}{\omega_j^2} \sum_{k=1,\ k\neq j}^{n} \rho_j\rho_k\alpha_j\alpha_k D_{jk} \right] |\hat{\psi}_{a_p,b_q}(\omega)|^2 \quad (37)$$

The parameter ρ_j is to be chosen depending upon the response quantity of interest. For example, this is equal to $\phi_i^{(j)}$ for the displacement response.

4. SPECTRAL MOMENTS

The statistics of a non-stationary response process may be estimated by obtaining the moments of the instantaneous PSDF of the process. In case of the PSDF corresponding to equation (37), the expression for the sth order moment becomes

$$\begin{aligned}\mu_s|_{t=b_q} &= \int_0^\infty \omega^s S_r^q(\omega)\,\mathrm{d}\omega \\ &= \sum_p \frac{K}{(\sigma-1)\pi} E[W_\psi z(a_p,b_q)]^2 \\ &\quad \times \sum_{j=1}^{n} \left[\left\{ \rho_j^2\alpha_j^2 + \sum_{k=1,\ k\neq j}^{n} \rho_j\rho_k\alpha_j\alpha_k(C_{jk}+D_{jk}) \right\} \mu_{s,j}^{p,D} - \frac{\mu_{s,j}^{p,V}}{\omega_j^2} \sum_{k=1,\ k\neq j}^{n} \rho_j\rho_k\alpha_j\alpha_k D_{jk} \right], \\ &\quad s=0,1,2,\ldots, \end{aligned} \quad (38)$$

where

$$\mu_{s,j}^{p,D} = \int_0^\infty |H_j(\omega)|^2\omega^s \chi_{[\frac{\pi}{a_p},\frac{\sigma\pi}{a_p}]}(\omega)\,\mathrm{d}\omega, \quad s=0,1,2,\ldots \quad (39)$$

and

$$\mu_{s,j}^{p,V} = \mu_{s+2,j}^{p,D} = \int_0^\infty |\omega H_j(\omega)|^2\omega^s \chi_{[\frac{\pi}{a_p},\frac{\sigma\pi}{a_p}]}(\omega)\,\mathrm{d}\omega, \quad s=0,1,2,\ldots \quad (40)$$

are the sth moments, respectively, for the displacement and velocity responses (corresponding to the pth band of energy) of the jth mode oscillator. In equations (39) and (40), $\chi_{[\,.\,]}$ is the indicator function, which takes unit value on the interval $[\,.\,]$ and is zero otherwise. Equation (38) can be rewritten as

$$\mu_s|_{t=b_q} = \sum_p \frac{K}{(\sigma-1)\pi} E[W_\psi z(a_p,b_q)]^2 \sum_{j=1}^{n} \rho_j^2\alpha_j^2\mu_{s,j}^{p,D}(1+\delta_{s,j}^p), \quad s=0,1,2,\ldots \quad (41)$$

where

$$\delta_{s,j}^p = \sum_{k=1,\ k\neq j}^{n} \frac{\rho_k\alpha_k}{\rho_j\alpha_j}\left[C_{jk} + D_{jk}\left(1 - \frac{\mu_{s,j}^{p,V}}{\omega_j^2\mu_{s,j}^{p,D}}\right)\right] \quad (42)$$

is the term accounting for the instantaneous cross-correlation of the jth mode with the remaining $n-1$ modes.[36] Significant simplification can be achieved by ignoring this term. However, this simplification will not necessarily have the same level of approximation as that in the SRSS (square-root-of-sum-of-squares)

technique of response spectrum superposition where the ignored terms pertain to the cross-correlation between the different modal maxima.[36] In fact, it will be shown in the next section that this simplification leads to reasonably accurate results even when the natural frequencies of the structural system are closely spaced.

The first four moments, $\mu_{s,j}^{p,D}$, $s = 0, 1, 2, 3$, of the instantaneous PSDF are obtained in closed form on using complex partial fractions and on evaluating the real parts:

$$\mu_{0,j}^{p,D} = \frac{1}{8\omega_{dj}\zeta_j\omega_j^3}\left[\zeta_j\omega_j \ln\left[\frac{\left\{\left(\frac{\sigma\pi}{a_p}+\omega_{dj}\right)^2+\zeta_j^2\omega_j^2\right\}\left\{\left(\frac{\pi}{a_p}-\omega_{dj}\right)^2+\zeta_j^2\omega_j^2\right\}}{\left\{\left(\frac{\sigma\pi}{a_p}-\omega_{dj}\right)^2+\zeta_j^2\omega_j^2\right\}\left\{\left(\frac{\pi}{a_p}+\omega_{dj}\right)^2+\zeta_j^2\omega_j^2\right\}}\right] + 2\omega_{dj}\left\{\tan^{-1}\left(\frac{\frac{\sigma\pi}{a_p}+\omega_{dj}}{\zeta_j\omega_j}\right)+\tan^{-1}\left(\frac{\frac{\sigma\pi}{a_p}-\omega_{dj}}{\zeta_j\omega_j}\right) - \tan^{-1}\left(\frac{\frac{\pi}{a_p}+\omega_{dj}}{\zeta_j\omega_j}\right)-\tan^{-1}\left(\frac{\frac{\pi}{a_p}-\omega_{dj}}{\zeta_j\omega_j}\right)\right\}\right] \tag{43}$$

$$\mu_{1,j}^{p,D} = \frac{1}{4\omega_{dj}\zeta_j\omega_j}\left[\tan^{-1}\left(\frac{\frac{\pi}{a_p}+\omega_{dj}}{\zeta_j\omega_j}\right)-\tan^{-1}\left(\frac{\frac{\pi}{a_p}-\omega_{dj}}{\zeta_j\omega_j}\right)-\tan^{-1}\left(\frac{\frac{\sigma\pi}{a_p}+\omega_{dj}}{\zeta_j\omega_j}\right) + \tan^{-1}\left(\frac{\frac{\sigma\pi}{a_p}-\omega_{dj}}{\zeta_j\omega_j}\right)\right] \tag{44}$$

$$\mu_{2,j}^{p,D} = \frac{1}{8\omega_{dj}}\ln\left[\frac{\left\{\left(\frac{\sigma\pi}{a_p}-\omega_{dj}\right)^2+\zeta_j^2\omega_j^2\right\}\left\{\left(\frac{\pi}{a_p}+\omega_{dj}\right)^2+\zeta_j^2\omega_j^2\right\}}{\left\{\left(\frac{\sigma\pi}{a_p}+\omega_{dj}\right)^2+\zeta_j^2\omega_j^2\right\}\left\{\left(\frac{\pi}{a_p}-\omega_{dj}\right)^2+\zeta_j^2\omega_j^2\right\}}\right] + \frac{1}{4\zeta_j\omega_j}\left[\tan^{-1}\left(\frac{\frac{\sigma\pi}{a_p}+\omega_{dj}}{\zeta_j\omega_j}\right)+\tan^{-1}\left(\frac{\frac{\sigma\pi}{a_p}-\omega_{dj}}{\zeta_j\omega_j}\right) - \tan^{-1}\left(\frac{\frac{\pi}{a_p}+\omega_{dj}}{\zeta_j\omega_j}\right)-\tan^{-1}\left(\frac{\frac{\pi}{a_p}-\omega_{dj}}{\zeta_j\omega_j}\right)\right] \tag{45}$$

$$\mu_{3,j}^{p,D} = \frac{1}{4}\ln\left[\frac{\left\{\left(\frac{\sigma\pi}{a_p}-\omega_{dj}\right)^2+\zeta_j^2\omega_j^2\right\}\left\{\left(\frac{\pi}{a_p}+\omega_{dj}\right)^2+\zeta_j^2\omega_j^2\right\}}{\left\{\left(\frac{\sigma\pi}{a_p}+\omega_{dj}\right)^2+\zeta_j^2\omega_j^2\right\}\left\{\left(\frac{\pi}{a_p}-\omega_{dj}\right)^2+\zeta_j^2\omega_j^2\right\}}\right] \tag{46}$$

with $\omega_{dj} = \omega_j\sqrt{1-\zeta_j^2}$. It may be shown that the higher moments of the PSDF can be obtained by using the following recursive relationship:

$$\mu_{s,j}^{p,D} = \left(\frac{\pi}{a_p}\right)^{s-3}\frac{(\sigma^{s-3}-1)}{s-3} + 2\omega_j^2(1-2\zeta_j^2)\mu_{s-2,j}^{p,D} - \omega_j^4\mu_{s-4,j}^{p,D} \tag{47}$$

Using equations (43)–(46) together with equation (47) enables the closed-form evaluation of the spectral moments in equation (38). Estimation of the statistics of the ordered peak amplitudes from these moments

may be carried out with the help of the first passage formulation by Vanmarcke[28] and the order statistics approach by Gupta and Trifunac,[29] Gupta,[30] and Todorovska *et al.*[31] Brief details of such an approach as considered by Basu and Gupta[33] are given in the Appendix.

5. NUMERICAL EXAMPLE

The wavelet-based stochastic approach as formulated in this paper is illustrated by considering a fixed-base, torsionally coupled, seven-storey shear building with $n=21$. This building has the floor dimensions, 26×33 m for the bottom three stories and 20×25 m for the rest. The mass and the radius of gyration of each of the lower three floors are $1{\cdot}15\times 10^6$ kg and 12·02 m, while the upper four floors are of $0{\cdot}8\times 10^6$ kg mass and 9·24 m radius of gyration each. The storey stiffnesses in the X-direction (i.e. in the direction of the ground motion) for this building are $2{\cdot}21, 2{\cdot}70, 3{\cdot}10, 3{\cdot}42, 5{\cdot}00, 5{\cdot}24, 5{\cdot}36\times 10^6$ kN/m for the lower to the upper stories. The corresponding values in the Y-direction, respectively, are $1{\cdot}39, 2{\cdot}18, 2{\cdot}70, 3{\cdot}11, 4{\cdot}79, 5{\cdot}08, 5{\cdot}23\times 10^6$ kN/m. The static eccentricities in the X- and Y-directions, respectively, are 0·2 and 0·26 times the radius of gyration for the upper three floors, 0·24 and 0·26 times the radius of gyration for the next lower floor, and 0·34 and 0·50 times the radius of gyration for the lowermost three floors. Thus, the centres of stiffness are staggered, while the centres of mass are assumed to lie on a vertical axis. The natural frequencies of this building are computed to be 12·49, 14·63, 18·87, 29·21, 34·99, 42·74, 45·81, 55·81, 63·11, 67·53, 75·27, 77·27, 88·52, 91·23, 96·62, 100·39, 108·54, 113·41, 122·41, 129·43 and 156·98 rad/s. The damping ratio has been assumed to be equal to 0.05 in all the modes.

The example ground motion process is considered to be corresponding to the recorded motion at Pacoima dam site during the 1971 San Fernando earthquake. The process has been simulated by using the SYNACC program (see References 37–41), and twenty accelerograms have been generated from the Fourier spectra of the recorded motion with the hypocentral distance taken as 10 km. In the absence of the soil strata information, the group velocity curves corresponding to the Westmoreland, Imperial Valley site as in Lee and Trifunac,[38] have been used. The wavelet co-efficients have been calculated for the generated time-histories according to equation (1), for $j=-17$ to 4 and $i=1$ to 2047 with Δb taken as 0·02. The squares of these co-efficients have then been averaged over the ensemble to get the expected values, i.e. $E[W_\psi f(a_j,b_i)^2]$ and thus to characterize the input stochastic process.

Figures 1 and 2 show the expected floor displacement and storey shear profiles, respectively, for the example building and ground motion process. These are calculated from the proposed stochastic formulation as well as from the time-history analyses for the simulated ground motions, and are normalized with respect to the overall maximum value in the respective figures. The curves W-1 and W-3 in both the figures, respectively, correspond to the largest and the third largest response peaks obtained from the proposed formulation, while S-1 and S-3 correspond to the simulation results for the largest and the third largest peaks. The response profiles in both the figures are seen to compare reasonably well. Excellent matching is also obtained in case of the (normalized) root-mean-square (r.m.s.) profiles, e.g. see Figure 3 for the displacement response, and in case of the normalized integrated expected square of the displacement response at the top floor of the building, $E[\int_0^t x_7^2(\tau)\,\mathrm{d}\tau/\int_0^T x_7^2(\tau)\,\mathrm{d}\tau]$ as in Figure 4.

It has been pointed out in the previous section that neglecting the instantaneous cross-correlation terms may not lead to signficant errors in the response estimation. To investigate this in detail, we consider two more earthquake excitations: (i) recorded S00E component at El Centro site during Imperial Valley Earthquake, 1940, and (ii) synthetically generated motion for Mexico Earthquake, 1985, at Mexico City site (see References 34 and 35). The El Centro motion is broad-banded with energy between 0·2–5·0 s while the Mexico City motion is narrow-banded with predominant energy near 2·5 s period. Assuming the square of the wavelet co-efficients, $W_\psi f(a_j,b_i)^2$, obtained for these two motions to characterize the underlying ground motion processes, the largest displacement profiles corresponding to the formulation with and without correlation terms are obtained and compared in Figures 5 and 6. It is seen that the two profiles are in reasonably

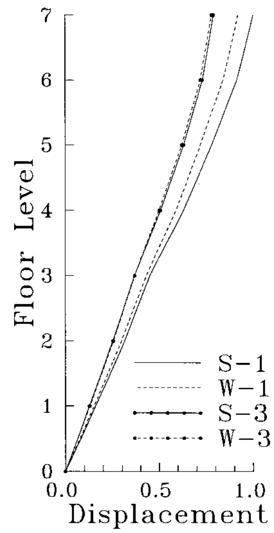

Figure 1. Comparison of displacement profiles from the proposed wavelet approach and simulation in case of the largest ($i = 1$) and third largest ($i = 3$) peaks

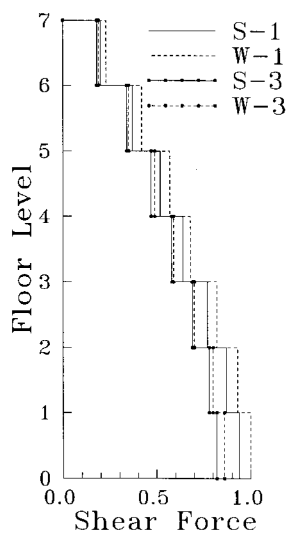

Figure 2. Comparison of shear force profiles from the proposed wavelet approach and simulation in case of the largest ($i = 1$) and third largest ($i = 3$) peaks

good matching for the El Centro case whereas in the case of the Mexico City motion, neglecting the cross-correlation terms appears to give significantly large errors. However, these errors are not as large as obtained by Gupta,[36] for the CQC-based response estimates. This suggests that approximating the instantaneous r.m.s. response by neglecting the cross-correlation terms leads to much less errors than the inaccuracies incurred in neglecting/approximating the cross-correlation between the modal maxima. Larger errors in case of the Mexico City motion as seen in Figure 6 appear to be due to the fact that in this case, the building system is much stiffer to the ground motion. To study this aspect in greater detail and to see how the errors at the instantaneous level propagate to the final response, we consider two additional example building systems

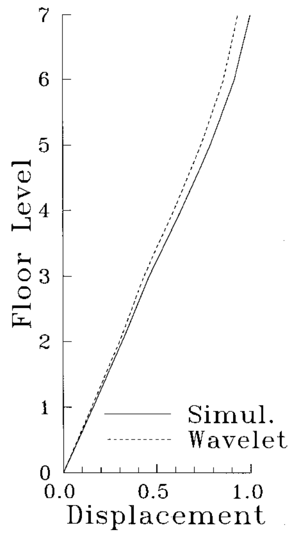

Figure 3. Comparison of RMS displacement profiles from the proposed wavelet approach and simulation

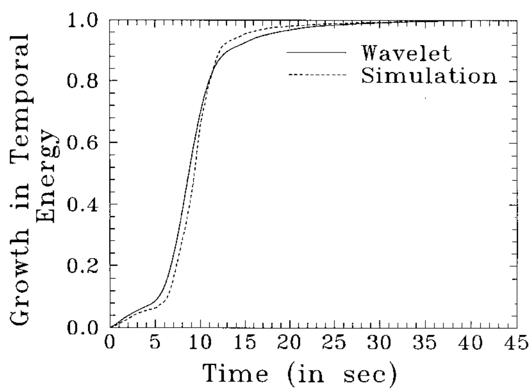

Figure 4. Comparison of growth in temporal energy from proposed wavelet approach and simulation

obtained by decreasing the storey stiffnesses of the example building, say Building #1, such that the natural periods of these buildings, #2 and #3, are increased by three and five times, respectively. Thus, Building #3 has its fundamental period close to the dominant period of ground motion. Figures 7–9 show the comparison of the temporal variations of the instantaneous r.m.s. displacement of the top floor with and without the cross-correlation terms in case of buildings #1, #2 and #3, respectively. It is clear from the three figures that the instantaneous values become more and more inaccurate as the building becomes stiffer to the ground motion. Further, since the level of error in Figure 7 is close to that observed in Figure 6, the inaccuracies in the estimation of the instantaneous response seem to affect the peak response as much.

6. CONCLUSIONS

A stochastic formulation based on wavelets has been developed for the seismic response of linear, classically damped, MDOF systems. This formulation takes into account the frequency non-stationarity in the input more realistically than the formulations based on the use of modulating functions. In comparison with stochastic

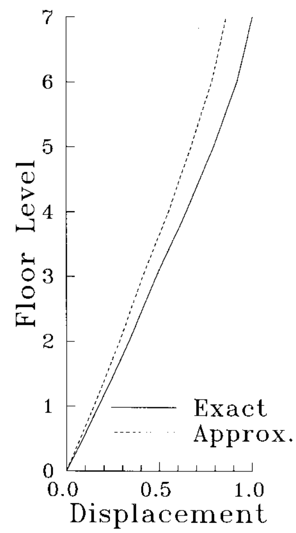

Figure 5. Comparison of the largest displacement profiles for exact (with correlation) and approximate (without correlation) analyses in case of the El Centro motion

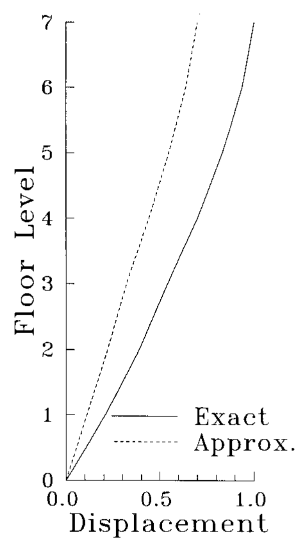

Figure 6. Comparison of the largest displacement profiles for exact (with correlation) and approximate (without correlation) analyses in case of the Mexico City motion

time-history simulations, this formulation has the advantage of providing frequency characteristics in form of time-dependent PSDF of the response, besides the temporal characteristics like ordered peak amplitudes, with much less computational effort. Closed-form expressions of the instantaneous spectral moments should facilitate the computations. It is possible to ignore the cross-correlation terms in the proposed formulation provided the structural system is not too stiff to the ground motion. It has been shown through a numerical study that the formulation can estimate the different ordered peak responses reasonably well. However, for this formulation to be of widespread utility to the engineers, the characterization of the ground motions has to be done through wavelet co-efficients instead of Fourier or response spectra.

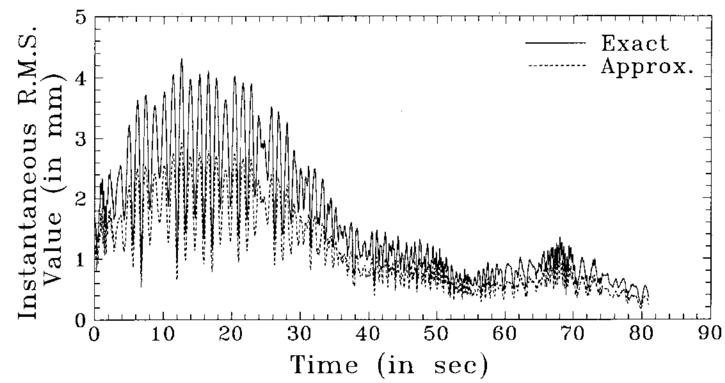

Figure 7. Comparison of the instantaneous RMS profiles for exact (with correlation) and approximate (without correlation) analyses in the case of building # 1

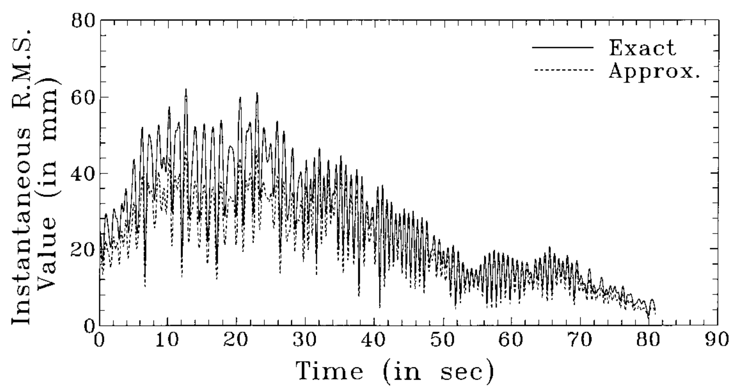

Figure 8. Comparison of the instantaneous RMS profiles for exact (with correlation) and approximate (without correlation) analyses in the case of building # 2

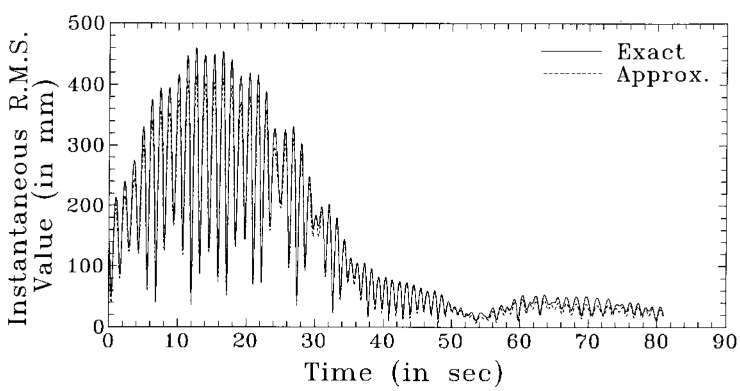

Figure 9. Comparison of the instantaneous RMS profiles for exact (with correlation) and approximate (without correlation) analyses in the case of building # 3

APPENDIX

The largest peak statistics in a non-stationary process $x(t)$, with duration T can be calculated based on the first-passage formulation by Vanmarcke.[28] This formulation gives the probability that the process $|x(t)|$ remains below the level x during the time interval $(0, T)$:

$$P_T(x) = \exp\left[-\int_0^T \alpha(t)\,\mathrm{d}t\right] = \exp\left[-\sum_q \alpha(t)|_{t=b_q}\Delta b\right] \tag{48}$$

where

$$\alpha(t) = \frac{\Omega_q}{\pi}\frac{1-\exp(-\sqrt{\pi/2}\lambda_{qe}(t)\frac{x}{\sigma_q})}{1-\exp(-\frac{x^2}{2\sigma_q^2})}\mathrm{e}^{-x^2/2\sigma_q^2} \tag{49}$$

is the rate parameter with $\lambda_{qe} = \lambda_q^{1+b}$ and $b = 0{\cdot}2$. Further, σ_q, λ_q and Ω_q are defined by

$$\sigma_q = \sqrt{\mu_0|_{t=b_q}} \tag{50}$$

$$\Omega_q = \sqrt{\frac{\mu_2|_{t=b_q}}{\mu_0|_{t=b_q}}} \tag{51}$$

$$\lambda_q = \sqrt{1-\frac{\mu_1^2|_{t=b_q}}{\mu_0|_{t=b_q}\mu_2|_{t=b_q}}} \tag{52}$$

The first-passage problem in its present form cannot predict the statistics of the higher-order peak responses and hence, an equivalent stationary process such that it has same ratios of the largest to the higher-order peaks as in the non-stationary process (at least for the first few order of peaks) is considered. This equivalent process should also have the same characteristics as that exist in the near vicinity of the largest peak in the non-stationary process. Thus, the duration T^* of the equivalent process is defined as

$$T^* = \int_0^T v_0(t)\,\mathrm{d}t/v_0(b_m) \tag{53}$$

where $v_0(.)$ $(=\Omega_q/2\pi)$ is the rate of zero crossings. In equation (53), b_m denotes the time-instant of occurrence of the largest mean-square response in the non-stationary process. The probability density function for the ith largest peak amplitude of the modulus of the equivalent process is given as[29–31]

$$\tilde{q}(\eta) = i\binom{\tilde{N}}{i}[1-\tilde{P}(\eta)]^{\tilde{N}-i}\tilde{P}(\eta)^{i-1}\,\tilde{p}(\eta) \tag{54}$$

In equation (54),

$$\tilde{p}(\eta) = \frac{1}{\sqrt{2\pi}}\left[2\varepsilon_m\mathrm{e}^{-\eta^2/2\varepsilon_m^2} + (1-\varepsilon_m^2)^{1/2}\eta\mathrm{e}^{-\eta^2/2}\int_{-\eta(1-\varepsilon_m^2)^{1/2}/\varepsilon_m}^{\eta(1-\varepsilon_m^2)^{1/2}/\varepsilon_m}\mathrm{e}^{-x^2/2}\,\mathrm{d}x\right]; \quad \eta \geqslant 0, \tag{55}$$

is the probability density function for the maxima, $\tilde{P}(\eta) = \int_\eta^\infty \tilde{p}(u)\,\mathrm{d}u$ is the cumulative probability,

$$\tilde{N} = \frac{T^*}{2\pi}(1+\sqrt{1-\varepsilon_m^2})\sqrt{\frac{\mu_4|_{t=b_m}}{\mu_2|_{t=b_m}}} \tag{56}$$

is the total number of expected peaks and

$$\varepsilon_m = \sqrt{1 - \frac{\mu_2^2|_{t=b_m}}{\mu_0|_{t=b_m}\mu_4|_{t=b_m}}} \tag{57}$$

is an instantaneous band-width parameter at the time-instant b_m. Further, the r.m.s. value of the equivalent stationary process is assumed such that the largest peaks in the non-stationary and its equivalent processes are same. The higher-order peak estimates are obtained using this 'equivalent' r.m.s. value and the probability function as in equation (54).

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